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ABSTRACT

An approach is described that reveals the hierarchical test structure (HTS) based on the cognitive demands of the test items, and conducts a linear trait modeling by using the HST elements as item difficulty components. This approach, referred to as the Hierarchical Latent Trait Approach (HLTA), employs an algorithm that allows all test items to be allocated to hierarchically ordered levels and defines the HTS. The HLTA is allied to data from two tests with samples of 47 and 49 undergraduate students in a statistics class. The following steps are followed: (1) determination of the cognitive operations required by the test items; (2) determination of the HTS; (3) linear trait modeling with (L,I,S) cognitive information components; (4) multiple regression analysis for (L,I,S) prediction of the item difficulty; and (5) item clustering. The HLTA provides both quantitative and qualitative information about parameters and relations of main interest in test analysis. Four tables and two figures illustrate the analysis. (Contains 8 references.) (SLD)

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Hierarchical Latent Trait Approach in Test Analysis

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FRAMEWORK

The present paper is related to the development of cognitive approaches in test analysis. Its main goal is to describe an approach revealing the **hierarchical test structure (HST)**, based on the cognitive demands of the test items, and conducting a linear latent trait modeling by using the **HST** elements as item difficulty components. To better define this approach, referred to here as **Hierarchical Latent Trait Approach (HLTA)**, let me define in more detail the concept of **HTS** in the sense it is used here.

Assuming the cognitive operations (**CO**) required by the test items have been established, we say that a given (i-th) item is "inferior" to another (j-th) item only if the set of cognitive operations required by the i-th item (CO_i) is a part of the set of cognitive operations required by the j-th item (CO_j), i.e. $CO_i \subset CO_j$. Concomitantly, the j-th item is said to be "superior" to the i-th item. In this sense, we say that i-th and j-th items are (cognitively) related in hierarchical order. The following algorithmic rule allows us to allocate all test items to hierarchically ordered levels and, as a matter of fact, defines the **hierarchical test structure (HTS)**:

HTS rule: "Level L_k follows level L_{k-1} in the increasing hierarchical order only if any item allocated to level L_k is "superior", as defined above, to at least one item allocated to level L_{k-1}."

Thus, the **HTS** is objectively determined by the above rule and any (say, i-th) test item is characterized by three components: L_i - level on which the item was allocated, I_i - number of items "inferior" to the item, and S_i - number of items "superior" to the item. The **HTS** can be also thought of as a "tree-type" graph diagram where all points (items) are located at hierachically ordered levels and an "arrow" goes from a given (i-th) point to another (j-th) point only if the item represented by the j-th point is "superior" to the item represented by the i-th point. In this sense, S_i will be the number of "arrows" that go from the i-th item and I_i will be the number of "arrows" that enter the i-th item in the **HTS** graph . Obviously, $I = 0$ for all items located at the lowest **HTS** level and $S = 0$ for all items located at the highest **HST** level (see fig.1).

The next step in the description of the Hierarchical Latent Trait Approach (**HLTA**) is related to the use of the **(L,I,S)** cognitive information components, as defined above, for predicting and analyzing the difficulty parameter of the test items. As a type of model, the model used here is a linear logistic latent trait model (Fischer, 1973; Embretson, 1984):

$$(1) \qquad P(x_{ij} = 1 | \theta_j, \eta_k, a) = \exp[\theta_j - (\sum_k c_{ik}\eta_k + a)]/\{1 + \exp[\theta_j - (\sum_k c_{ik}\eta_k + a)]$$

where: x_{ij} = the response of person j to item i;

θ_j = ability for person j;

η_k = the difficulty of complexity factor k (part of the total item difficulty);

c_{ik} = the coefficient of η_k in the linear representation of the item difficulty by the set of complexity factors $\eta_1, \eta_2, .., \eta_m$;

a = a normalization constant.

In particular, the **HLTA** idea is to use the **(L,I,S)** cognitive information components as complexity factors in model (1), i.e.

η_1 = **L** (the item level in **HTS**); the respective coefficient c_{i1} will be the numerical order of the level in the hierarchical test structure (**HTS**).

η_2 = **I** (the "input" cognitive information of the item); the c_{i2} coefficient will show the number of items "inferior" to the i-th item.

η_3 = **S** (the "output" cognitive information of the item) ; the c_{i3} coefficient will show the number of items "superior" to the i-th item.

The item difficulties calculated by using model (1) with **(L,I,S)** complexity components will be referred to here as **HLTA item difficulties**. A Fortran program, **LINLOG** (that was kindly provided by Dr.Susan Embretson of the University of Kansas) was used for the calculation of the η_k values, the **HLTA** item difficulties, and the regression of the Rasch item difficulties on the **HLTA** item difficulties.

The **HLTA** as presently applied also includes some other validation procedures such as multiple regression analysis, with **(L, I, S)** predictors of the item difficulty, and cluster analysis described bellow in more details.

METHODS:

Data source: Two tests (midterm and final) in a statistics course for undergraduate students in SIUC, Spring, 1994.

Student sample size: 47 for the midterm test and 49 for the final test.

Item sample size: 20 items on the midterm test and 24 items on the final test.

Item domain: reproduced (with adaptations) from the set of examples and exercises in the course textbook (Moore, D. & McCabe, G., 1993).

PROCEDURES

1. Determination of cognitive operations required by the test items:

On the basis of expert analysis, nine cognitive operations (**CO**) were inferred from the process of solving a set of items validated in consistence with domain parameters, educational goals, etc. These operations were defined at a suitable level of generality appropriate for the type of items. In the **CO** description, given bellow, **CRP** stands for "**Concept, Rule, or Principle**":

CO_1 : Straight **CRP** identification from a set of given options.
CO_2 : **CRP** identification based on straight inference from a verbal interpretation.

CO_3 : **CRP** identification based on inference from implicit contextual information.
CO_4 : Usage of **CRP** for straight inference, justification, or explanation of statement, decision, situation, or phenomenon.
CO_5 : Straight application of simple routine procedure (usage of statistical formula, table, histogram, diagram, etc.).
CO_6 : Solving familiar algorithm-type problems (outlining the design of familiar types of experiments, testing familiar types of hypotheses about means, proportions, etc.)
CO_7 : Solving familiar non algorithm-type problems (usage of familiar and practiced procedures, after an appropriate interpretation and/or classification processing).
CO_8 : Solving unfamiliar "jointing-type" problems (selection of appropriate rules, procedures, etc., and "jointing" them in a solving method on the base of logical relations).
CO_9 : Solving unfamiliar "analysis-type" problems (solving enhanced CO_8 -type problems, discerning patterns and/or tendencies, evaluation processing, etc.).

The results of analysis showing which of the cognitive operations CO_1, ..., CO_9 are required by the respective test items can be summarized in a two-way table with 1 in the (i,j) cell if the CO_j cognitive operation is required by the i-th item (otherwise, 0 in the cell) . This table, referred to here as "**I-CO**" (Items by Cognitive Operations) facilitates the making of the **HTS matrix** defined in the next procedure.

2. Determination of the HTS (Hierarchical Test Structure):

A first step in this procedure is to make a two-way table (referred here as to **HTS matrix**) with **N** rows and **N** columns, where **N** is the number of test items. We put a value of 1 (one) in the cell (i,j) if the set of cognitive operations (**CO**) required by the j-th item represents a part of the set of cognitive operations (**CO**) required by the i-th item, i.e. i-th item is "superior" to the j-th item in the sense defined at the beginning. Otherwise we leave the (i,j) cell empty ; (i, j = 1, 2, ..., N). In other words, we put 1 in the cell "intersection" of i-th row and j-th column of the HTS matrix only if the i-th item requires all cognitive operations required by the j-th item plus at least one more cognitive operation. The **HTS** matrix for the midterm test items (from the data source used in the present study) is illustrated by Table 1-M.

Insert **Table 1-M** about here

Similarly, Table 1-F is the **HTS** matrix for the final test items.

Insert **Table 1-F** about here

The earlier **defined HTS rule,** when applied to the **HTS matrix,** determines the **HTS** (Hierarchical Test Structure). Fig.1 illustrates the **HTS** inferred from Table1-M and represented in the form of a "tree-type" graph with arrows connecting any two items which are in a hierarchical cognitive dependence (not all the arrows are given in fig.1, but they can be directly "restored" from the **HTS** matrix.). It can be seen that the **HTS** for the midterm test contains five hierarchically ordered levels, from the lowest (L_1) to the highest (L_5) level.

Insert **fig. 1** about here

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3. Linear latent trait modeling with (L, I, S) complexity components:

In order to use the linear latent trait model (1) with complexity components $\eta_1 = L$, $\eta_2 = I$ and $\eta_3 = S$, we have to determine the respective coefficients c_{i1}, c_{i2}, and c_{i3} for all items (i =1, 2, ..., N). This is to be done by using the **HTS** information. For example, from fig.1 we can see that item # 4 is located at the fourth level (**L_4**) which means that **L** = 4 for this item, i.e. $c_{41} = 4$. Further, we can see four arrows ending in item # 4 (or, differently, four 1's in the fourth row of Table1-M). This means that there are four items "inferior" to item # 4 and **I** = 4 for this item, i.e. $c_{42} = 4$. Finally, there are two arrows starting from item # 4 (or, differently, two 1's in the fourth column of Table1-M). This means that there are two items "superior" to item # 4 and S = 2 for this item, i.e. $c_{43} = 2$.

With **(L,I,S)** coefficient information for all items, we use the **LINLOG** program for the calculation of η_1, η_2, η_3, the **HLTA** item difficulties, and the Rasch item difficulties. The data fit to the Rasch model was tested by the MICROSCALE program (B.Wright & J. Linacre, 1984).

4. Multiple regression analysis for (L, I, S) prediction of the item difficulty:

The dependent variable in this multiple regression analysis is the Rasch item difficulty predicted by the complexity components **L, I,** and S with values calculated in Procedure 3. Of primary interest here is the overall contribution of the three predictors for the variability of the Rasch item difficulties as well as their partial contribution to this variability.

5. Item clustering:

The item clustering is used for an additional HTS validation. The starting matrix $\| x_{ij} \|$ in this procedure has **n** rows (**n** students) and N columns (N items) with $x_{ij} = 1$ if i-th student has answered j-th item correctly, otherwise $x_{ij} = 0$. The inferred matrix of similarity $\| s_{jk} \|$ has N rows and N columns with s_{jk} representing the level of similarity between j-th and k-th items, i.e. what proportion of all N values (1 or 0) with the same position number in j-th and k-th columns of the matrix $||x_{ij}||$ are equal. Then, on the basis of the similarity matrix $||s_{jk}||$, all test items are grouped in clusters with respective levels of similarity. Technically, one can use different cluster programs (on SAS, SPSS, SYSTAT, etc.) with $||x_{ij}||$ as an input data matrix.

RESULTS

Table 2-M is a table-type representation of the **HTS** (Hierarchical Test Structure) for the midterm test from our data source. It is a different way to represent the information from Table1-M and fig.1. The difference is that Table 2-M contains the Rasch item difficulties but does not show to which items any given item is "inferior" or "superior", i.e. the **HTS** "arrows" are missing in Table 2-M.

Insert **Table 2-M** about here

In Table 2-M the Rasch difficulty of any item is given in parentheses following the number of the item. The **I** and **S** values, given at the second row in each cell, show the number of items "inferior" (resp. "superior") to the correspondent item from the cell. One can see that items 1, 10, 9, and 20 are located at the lowest **HTS** level (**L** = 1) and their Rasch difficulties vary from -1.53 to -1.31 in the logit scale. Items 3, 17, 11, 15, 6, and 13 are located the second **HTS** level (**L** = 2) and their Rasch difficulties vary from -.99 to .03. Items 5, 12, 16, and 7 are located at the third **HTS** level (**L** = 3) with Rasch difficulties varying from .14 to .87. Items 4, 8, and 14 are located at the fourth **HTS** level with the range of difficulties from .61 to 2.12. Finally, items 18 and 19 are located at the highest **HTS** level (L = 5) with the Rasch item difficulty of this level reduced to a single value of 2.39. The explanation of this result is that items 18 and 19 require the same cognitive operations. One can also see that the Rasch item difficulties increase with the increase of the **HTS** level. The difficulty intervals taken successively from all **HTS** levels "cover" virtually the total range from the lowest (-1.53) to the highest (2.39) Rasch item difficulty.

The results from Table 2-F concern the other ("final"), but they should be interpreted in exactly the same way as those from Table 2-M. One can see the same behavior of the relation "Rasch item difficulties - **HTS** levels" as was the case in Table 2-M.

Insert **Table 2-F** about here

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According to model (1), the HLTA diffficuly b_i of some (i-th) item is calculated as follows:

(2) $$b_i = c_{i1}\eta_1 + c_{i2}\eta_2 + c_{i3}\eta_3 + a$$

For the midterm test we obtained :$\eta_1 = .8738323$, $\eta_2 = .0792703$, $\eta_3 = .0534297$, and normalization constant $a = -2.78561$. On the other hand, from the **HTS** we know the **L, I,** and **S** values of the i-th item, i.e. we know c_{i1}, c_{i2}, and c_{i3}, respectively. Hence, the calculation of the **HLTA** item difficulties b_i is simple enough when using equation (2). Still for the midterm test, the correlation between the **HLTA** difficulties and the Rasch difficulties of the respective 20 items was found to be $R = .963$. This extremely high correlation is consistent with the $R^2 - value$ that was found from the multiple regression analysis used for the prediction of the Rasch difficulties by the complexity components **L, I,** and **S:** $R^2 = .9305$. The forward stepwise regression analysis showed that most of this high prediction is due to the **L**-component (with $R^2 = .9167$). The partial R^2 (the "over and above" prediction) was found to be .0099 and .0039 for the other two components, **I** and **S,** respectively.

Similarly, for the final test we obtained: $\eta_1 = .072252$, $\eta_2 = .061925$, $\eta_3 = -.105452$, and normalization constant $a = .01009$. The respective HLTA difficulties, calculated on the basis of equation (2), highly correlated with the Rasch difficulties for the respective 24 items: $R = .948$. As in the previous case, this result was consistent with the $R^2 - value$ found from the multiple regression analysis for the prediction of Rasch difficulties by the complexity components **L, I,** and **S:** $R^2 = .899$. Here, again, the most important prediction factor was the **L**-component with its $R^2 = .85$. The "over and above" contribution of the other two component, **S** and **I,** was .0274 and .0212, respectively.

The result from the item clustering (Procedure 5) is illustrated in fig. 2 for the midterm test. There are four clusters of items at the highest level of similarity (1.00). If we take the items of any of these clusters we will see that they are located at the same **HTS** level (Table 2-M) and their difficulties are the same or very close. For example, the items of the cluster {#18, #19} are located at the same **HTS** level (**L**=5) and they have the same difficulty (2.39). The items of the cluster {#7, #16} are located at the same **HTS** level (**L**=3) and their difficulties are .87 and .73, respectively. The items of the cluster {#9, #20} are located at the lowest HTS level (**L**=1) and have the same difficulty (-1.31). Finally, the items of the cluster {#11, #15} are located at the same **HTS** level (**L**=2) and have the same difficulty (-.69). The fifth cluster includes at a very high level of similarity (.88) all items except the two most difficult items {#18, #19) and the easiest item #2. The last cluster combines the fifth cluster and item #2 at level of similarity .77 which is still very high.

Insert **fig. 2** about here

The above interpretation of fig. 2 suggests a high similarity of the item response patterns. This can be explained by the relatively high homogeneity of the student samples which is typical for most university courses. Further, the clustering of the items is consistent with their allocation at the **HTS** levels and their difficulties which is important for the **HLTA** validation.

DISCUSSION

The Hierarchical Latent Trait Approach (**HLTA**), as presently defined and illustrated, provides both quantitative and qualitative information about parameters and relations of main interest in test analysis. The "new thing" with the **HLTA** is the **HTS** cognitive model with four characteristics of the test items : **L** (HTS level), **I** ("input" cognitive information), **S** ("output" cognitive information), and **(I-S)R** ("inferiority"/"superiority" relations) given, for example, by "arrows" between some items in the **HTS** graph representation (see fig. 1).

The terminology of "hierarchically ordered levels of test items" is used in some cognitive models for test analysis but in a sense quite different from this of the **HTS** model. For example Gitomer & Rock (1993) define the following three hierarchical levels: **Level 1** = Task recognition + application of simple rules; **Level 2** = Insight + application of simple rules; **Level 3** = Insight + production + application of simple rules. In this type of models the items are categorized in a fixed number of levels on the base of subjective expert judgments, whereas the HTS rule objectively infers the number of levels and the item allocation from the **HTS** matrix (as defined at the beginning). Moreover, the **HTS** model provides the above mentioned item characteristics **I, S,** and **(I-S)R** which are not inherent in the other models. In this sense, the **HTS** model is unique in its cognitive characteristics and provides quantitative coefficients of **L, I,** and S that makes possible their use as complexity components in the linear latent trait model (1). Although in both midterm and final tests the **I** and **S** components were almost negligible in comparison with the **L**-component for the prediction of the item difficulties, we still recommend their use in the HLTA expecting that they may play more significant role in some different type of testing situations.

The **HLTA** validation starts with expert-based judgments about the appropriateness of the cognitive operations in the context of content domain, testing purposes, etc. In this particular **HLTA** illustration we defined nine cognitive operations ($CO_1, ..., CO_9$) on the basis of which the **HTS** was objectively determined. Further, the **LINLOG** program tests in two different ways the adequacy of the complexity components **L, I,** and **S,** inferred from the **HTS** for the linear latent trait modeling: (A) a log likelihood χ^2 test for differences in goodness of fit between model (1) and the Rasch model ; (B) Pearson correlation between the Rasch item difficulties and the item difficulties estimated by model (1). What we obtained, for example, from the midterm test data was respectively: (A) $\chi^2 = 18.51$ with $df = 17$ and (B) $R = .963$. From the final test data we obtained: (A) $\chi^2 = 69.35$ with $df = 21$ and (B) $R = .847$. Interpreting these results we can say that the χ^2 value, for both midterm and final tests, shows no significant difference in goodness of fit between model (1) and the Rasch model. At the same time the correlation coefficient R shows

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extremely good prediction of the Rasch item difficulties from the complexity components **L, I,** and **S** for both the midterm and final tests.

Despite the relatively small student sample sizes, we used the Rasch item difficulties for the purposes of the **HLTA** validation, relying on the replication of the **HLTA** with midterm and final tests. In both cases the data fit the Rasch model according to the "outfit/infit" rules (Wright & Linacre 1984, Chapter 4, pp. 1 - 34). Moreover, for both midterm and final tests we obtained an additional **HLTA** validation by the consistency of the results from all **HLTA** procedures - **HST** developing, linear latent trait modeling, multiple regression analysis and cluster analysis.

In conclusion we can say that the results from the **HLTA** procedures provide different pieces of information for making diagnostic decisions about item characteristics, students' abilities and cognitive processes required to solve problems within a test. As presently applied, the **HLTA** is related to the testing of achievement on cognitive items. However, if, instead of cognitive operations, some characteristics influencing the score on personality items are defined, it can be used for obtaining both quantitative and qualitative information from a personality test.

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Table 1-M

HTS matrix for the midterm test: (i, j)=1 means that i-th item is "superior" to j-th item
I = number of items "inferior" to i-th item ; **S** = number of items "superior" to j-th item

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	**I**
1																					**0**
2																					**0**
3	1																				**1**
4	1		1		1								1								**4**
5	1												1								**2**
6										1											**1**
7	1								1		1		1		1					1	**6**
8	1		1		1		1		1		1	1	1		1	1				1	**11**
9																					**0**
10																					**0**
11	1								1											1	**3**
12	1		1			1			1						1					1	**6**
13	1																				**1**
14	1		1		1	1				1			1								**6**
15	1								1											1	**3**
16	1								1		1		1		1					1	**6**
17	1									1											**2**
18	1		1	1	1	1	1		1	1	1	1	1	1	1	1	1			1	**16**
19	1		1	1	1	1	1		1	1	1	1	1	1	1	1	1			1	**16**
20																					**0**
S	**14**	**0**	**6**	**2**	**5**	**4**	**3**	**0**	**8**	**5**	**5**	**3**	**8**	**2**	**6**	**3**	**2**	**0**	**0**	**8**	

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Table 1-F

HTS matrix for the final test: (i,j)=1 means that i-th item is "superior" to j-th item
I=number of items "inferior" to i-th item; **S**=number of items "superior" to j-th item

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	**I**
1																									**0**
2																									**0**
3																									**0**
4																									**0**
5																									**0**
6	1	1	1	1	1					1		1	1			1	1			1					**11**
7	1	1	1	1	1	1					1	1								1	1				**10**
8	1	1	1	1	1	1			1	1		1	1			1									**11**
9	1	1	1	1	1	1				1	1	1					1			1					**11**
10																									**0**
11	1	1		1	1					1			1			1				1					**8**
12	1	1	1	1	1					1			1			1				1					**9**
13																									**0**
14						1	1		1	1								1		1	1				**7**
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	**23**
16													1												**1**
17	1	1	1	1	1					1										1	1				**8**
18	1	1	1	1	1	1	1				1	1	1			1	1		1						**13**
19						1			1	1										1	1				**5**
20																									**0**
21	1	1		1	1					1										1					**6**
22	1	1	1	1	1	1				1	1	1	1			1		1		1	1				**14**
23	1	1	1	1	1	1				1	1	1	1			1		1		1	1				**14**
24	1	1	1	1	1	1	1	1	1	1	1	1	1			1	1	1	1	1					**18**
S	**13**	**13**	**11**	**13**	**13**	**9**	**4**	**2**	**5**	**13**	**7**	**9**	**10**	**1**	**0**	**9**	**5**	**5**	**3**	**13**	**7**	**1**	**1**	**1**	

Graph form of the **HTS** (inferred from Table 1-M) with arrows drawn only for illustration of the relations and the parameters of item # 4 (**L**=4, **I**=4, and **S**=2)

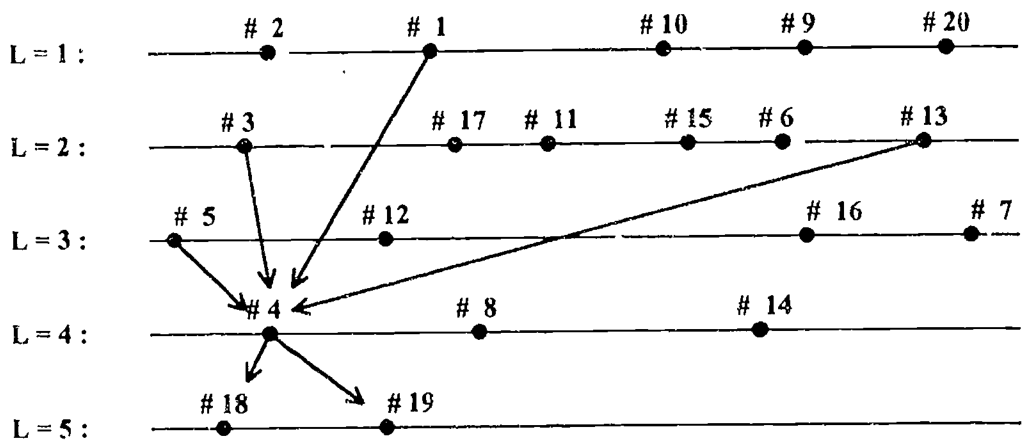

fig. 1

Table 2-M

Table form of the **HTS** for the midterm test (with the Rasch difficulties given in parentheses)

HTS level	Item #	Item #	Item #	Item #	Item #	Item #
L = 1	# **2** (-2.36) I=0 ; S=0	# **1** (-1.53) I=0 ; S=14	# **10** (-1.53) I=0 ; S=5	# **9** (-1.31) I=0 ; S=8	# **20** (-1.31) I=0 ; S=8	
L = 2	# **3** (- .99) I=1 ; S=6	# **17** (- .89) I=2 ; S=2	# **11** (- .69) I=3 ; S=5	# **15** (- .69) I=3 ; S=6	# **6** (- .38) I=1 ; S=4	# **13** (.03) I=1 ; S=8
L = 3	# **5** (.14) I=2 ; S=5	# **12** (.73) I=6 ; S=3	# **16** (.73) I=6 ; S=3	# 7 (.87) I=6 ; S=3		
L = 4	# **4** (.61) I=4 ; S=2	# **8** (1.67) I=11 ; S=0	# **14** (2.12) I=6 ; S=2			
L = 5	# **18** (2.39) I=16 ; S=0	# **19** (2.39) I=16 ; S=0				

ERIC

Table 2-F

Table form of the **HTS** for the final test (with Rasch difficulties given in parentheses)

Level	**Item #**	**Item #**	**Item #**	**Item #**	**Item #**	**Item #**	**Item #**	**Item #**
L= 1	# **1**(-2.06) I= 0;S=13	# **10**(-1.9) I=0; S=13	# **20**(-1.9) I=0; S=13	# **4**(-1.6) I=0;S=13	#**13**(-1.3) I=0;S=10	# **3** (-.86) I=0;S=11	# **5** (-.65) I=0;S=13	#**2** (-.55) I=0;S=13
L = 2	# **6** (-.25) I=1; S=9	#**21** (.03) I=6;S=7						
L=3	# **11** (.03) I=8 ; S=7	# **12** (.12) I=9; S=9	# **17** (.12) I=8 ; S=5					
L=4	# **6** (.22) I=11;S=9							
L=5	# **7** (.32) I=10;S=4	# **9** (.32) I=11;S=7						
L=6	# **19** (.41) I=5; S=3	# **8** (.61) I=11;S=2						
L=7	# **18** (.83) I=13;S=5							
L=8	# **14** (1.18) I=7; S=1	# **23**(1.44) I= 14;S=1	# **22**(1.58) I=14;S=1					
L=9	# **24**(1.73) I=18;S=1							
L=10	# **15**(2.08) I=23;S=0							

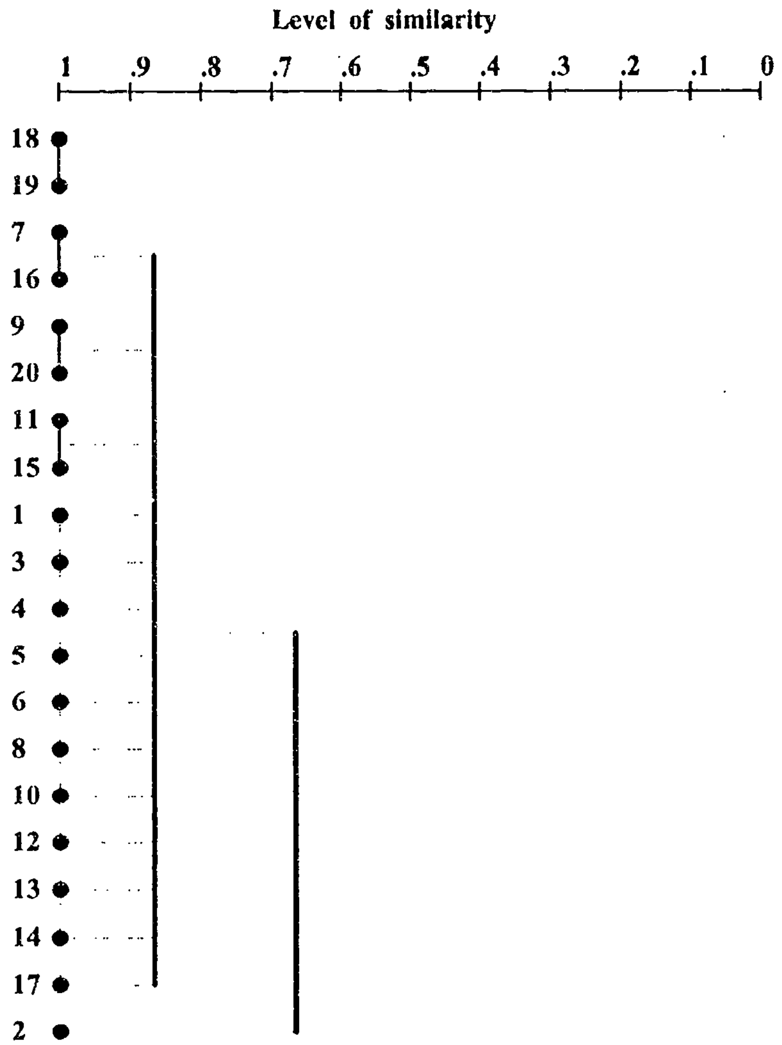
Level of similarity
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
18
19
7
16
9
20
11
15
1
3
4
5
6
8
10
12
13
14
17
2

fig. 2